WAVE-SEDIMENT INTERACTION ON A MUDDY SHELF

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Abstract: A simple unidirectional nonlinear model is presented for the study of the combined effect of mud-induced wave dissipation and 3-wave interactions on wave evolution over muddy environments. The model is used to simulate wave conditions typical for a muddy shallow shelf. Mud-induced long-wave dissipation introduces in the spectral cascade an energy sink which modifies 3-wave interaction behavior to produce net energy transport from short to long waves. Numerical experiments support the hypothesis that the process drains energy from high-frequency bands which would otherwise be unaffected by bottom dissipation mechanisms.

INTRODUCTION

The phenomenon of wave dissipation over muddy seabeds is of global interest. Such fine-grained shelf deposits occur worldwide, particularly in association with fluvial-marine dispersal systems. These low-gradient coasts can be impacted by major storms, and are often densely populated, such as the Mississippi Delta, the Ganges-Brahmaputra Delta, much of the Southeast Asian coast, and the European North Sea coast.

Waves have a strong impact on muddy beds. Catastrophic sea bed instabilities (submarine landslides, Sterling and Strohbeck, 1975) can be triggered by extreme storm waves in areas with high deposition rates and small slopes (between 0.1^{o} and 0.8^{o}, roughly 0.4 to 3%, Roberts et al. 1980). Intense resuspension (high mud concentrations extending meters above the previous mud-water interface) can occur when waves and currents exceed certain critical levels. Settling during the waning of storm can form a 20 to 50-cm thick bottom fluid mud layer with concentrations of the order of 20 to 100 kg/m^3.

Linear non-breaking wave analysis over muddy bottoms assumes that bottom-related dissipation is restricted to long waves (e.g. $\lambda \gg h$, with λ the wave length and h the water depth) which have significant near-bottom orbital velocities, while short waves propagate unaffected by bottom sediment type. Early field experiments have

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focused on the high dissipation rates of long waves (e.g. Forristall and Reece 1985), typically by deploying pairs of deep/shallow water observation stations along the probable path of long waves. Seabed motions have been monitored assuming the existence of a well defined, albeit soft, bottom. Short waves and mud fluidization effects were neglected.

Recent field observations show unexpected short-wave sensitivity to sediment type. Sheremet and Stone (2003) studied the effects of sediment type on wave propagation by comparing synchronous wave observations from two locations along the same isobath (5 m), one sandy and the other muddy. The locations were on a very mild sloping shelf (slope < 1/1000), and subject to nearly identical meteorological and offshore wave conditions. Comparable short-wave ($f > 0.2$ Hz) energy levels were recorded at the two sites when wind forcing was significant. However, during calmer periods, such as in the wake of a frontal passage, short waves dissipated much faster at the muddy site. Later, sediment monitoring devices (optical backscatter sensors) deployed at the same muddy site during Hurricane Claudette (Sheremet et al., 2004) showed strong spectrum-wide wave damping coinciding with the formation of a ~1-m thick fluid mud layer with sediment concentrations of over 50 kg/m^3.

The short-wave dissipation mechanism is unknown. Sheremet et al. (2004) hypothesize that nonlinear spectral transfer effectively couples short waves to the dissipative muddy bottom, resulting in a spectrum-wide influence of this bottom.

On a sandy shelf at the deeper end, nonlinear coupling of low- and high-frequency spectral bands in deep water occurs through 4-wave interactions, with net significant energy transfer over spatial scales of the order of hundreds of even thousands of wavelengths. As water becomes shallow, near-resonant 3-wave interactions dominate, with significant energy transfer over a few wavelengths (Elgar and Guza 1985; Agnon et al. 1993; Herbers et al. 1994, 1995; Kaihatu and Kirby 1995; Agnon and Sheremet 1997, Kaihatu 2001; Sheremet et. al 2003; and many others). The stronger coupling changes the shape of the individual waves and generates low frequency (long) infragravity waves. Strong phase correlations build up between spectral components, and the wave field is no longer Gaussian.

Nonlinear processes should be important over a typical wide (10 km) and shallow (15-20 m depth) muddy shelf. However, the role of wave nonlinearity in the evolution of wave spectra in environments characterized by strong (frequency dependent) damping has not been studied. Observed short-wave dissipation suggests that nonlinear energy transfer within the wave spectrum may be important, perhaps providing the coupling between the short- and long-wave spectral bands, allowing energy to flow toward long waves, where it can be efficiently dissipated via direct wave-bottom interaction. Observations of soliton trains in shallow muddy water (Wells, 1978), and of significant infragravity ($f < 0.05$ Hz) wave energy on the Louisiana shelf during storms, suggest that wave nonlinearities are active in the presence of high suspended sediment concentrations.

Here, we examine the effects of frequency dependent dissipation on nonlinear wave evolution by coupling numerical simulations of mud-induced wave dissipation and of nonlinear wave evolution dominated by 3-wave interactions.

MUD-INDUCED WAVE DISSIPATION

The strong wave dissipation effect induced by cohesive sediments is well known. Over mudbanks off the coast of Kerala, India, Mathew et al. (1995) report 95% reduction in wave energy of 7 s waves over a distance of 1.1 km. In the laboratory, Gade (1957) observed 80% wave energy loss over only 2.6 wavelengths for a mud layer thickness of about 1.3 times the wave boundary layer thickness. Dalrymple and Liu (1978) developed a linear formulation for waves propagating over a 2-layer fluid with a viscous mud layer. The mechanism of wave attenuation is the forcing of an interfacial wave by pressure work on the lower layer. Interfacial waves increase in amplitude with the thickness of the mud layer, and slightly phase-lag the surface wave. Their simulations agreed well with Gade's observations. Jiang and Mehta (1996) examined the second order bound wave theory using the perturbation approach for a viscoelastic mud to calculate mass transport. The model was reproduced data collected in the laboratory.

The dissipation of a monochromatic wave of frequency *f* is described by a linear equation describing spatial evolution of the wave amplitude *a*

$$\frac{d}{dx}a(x,f)=-\kappa(f)a(x,f), \qquad (1)$$

where the dissipation coefficient $\kappa(f)$ is a strong function of the frequency. Here, mud-induced wave dissipation (Figure 1) is estimated using the approach developed for monochromatic waves over a viscous mud layer by Dalrymple and Liu (1978) and extended by Jiang (1993), and Jiang and Mehta (1995, 1996) to account for non-Newtonian mud dynamics. The model estimates κ under conditions likely to be encountered in the field. Figure 1a shows the frequency dependence of dissipation. The typical shape of $\kappa(f)$ is bell-like, with a peak frequency and a sharp decrease in dissipation at higher and lower frequencies. The actual position of the peak and decay at high/low frequencies depends on multiple factors: the depth of the clean water column, the thickness of the fluid mud layer, the type and composition of the clay, and fluid-mud density. The dependence on some of these parameters (e.g. clay composition) is difficult to formalize. In the simulations presented here, the parameters were given values deemed to be representative of a wave-dominated muddy environment (e.g. Louisiana shelf, Sheremet et al. 2004). The thickness of the mud layer is 0.3 m, water depth is 5 m, and mud density of 1,130 kg/m^3. The fluid mud layer is approximated a viscous fluid, with a mud dynamic viscosity of 10^3 Pa s.

Due to the exponential attenuation with depth of wave motion, wave-bottom interaction weakens at high frequencies and the depth of the water layer limits the bottom-induced dissipation in the short wave band. In Figure 1, linear waves above about 0.5 Hz are not attenuated. For frequencies that do reach the seafloor, the attenuation drops off at frequencies below about 0.14 Hz, where the wavelength is much longer than the mud layer thickness. The result is an attenuation maximum (Figure 1a) and transmission minimum (Figure 1b) at about 0.14 Hz. Both the κ-curve and transmission show a steep variation for frequencies between 0.14 and 0.4 Hz. Within a certain range, higher density suspensions will produce higher dissipation rates due to increased viscosity.

As mud density increases beyond a certain critical value, the bottom behaves essentially as a rigid solid and dissipation decreases. Rheological properties of mud suspensions are usually strong functions of density (Jiang and Mehta 1995; Lee and Mehta 1997; and many others).

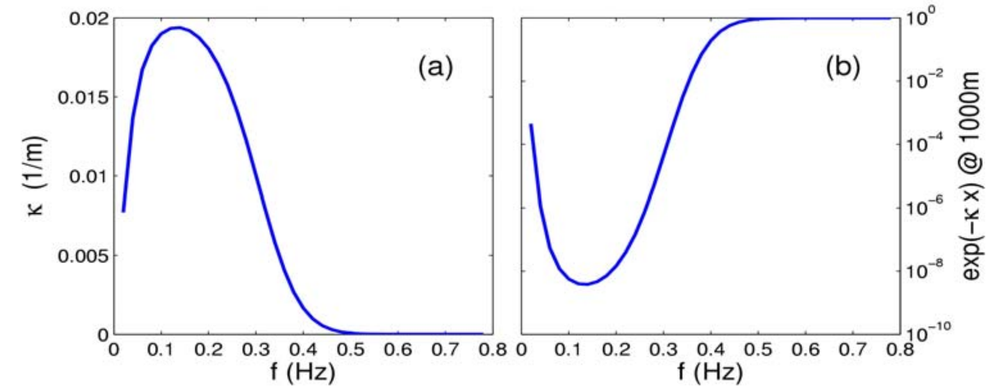

Fig. 1. (a) attenuation coefficient and (b) amplitude transmission factor at 100 m (fraction of initial amplitude preserved after a propagation distance of 1,000 m), versus frequency. The curves are calculated using the Jiang and Mehta (1996) model. The water depth is 5 m, fluid mud layer depth is 30 cm, fluid mud density is 1,130 kg/m^3, and mud viscosity 10^3 Pa s.

Several wave dissipation curves are discussed in Jiang (1993) for different experimental settings. He also noted that there seemed to be no measurable correlation between wave amplitudes and wave dissipation.

NONLINEAR WAVE DYNAMICS

The problem of nonlinear damped waves has been approached previously with noteworthy results. Miles (1976) investigated the equations for superharmonic resonance. Wersinger et al. (1980a,b) showed that three near-resonant modes can exhibit chaotic behavior if two modes are amplified while one is dissipated. However, there are apparently no formulations for nonlinear surface/interface wave propagation over a fluid mud layer. Here, we assume that surface wave triads can be qualitatively described by introducing dissipation into existing nonlinear wave models. To include cases when not all the surface waves are long waves (e.g. short-wave groups), we use the fully dispersive deterministic mild slope equation developed by Agnon et al. (1993), and Agnon and Sheremet (1997, 2000).

The deterministic evolution equations accounting for equations 3-wave interactions are:

$$\frac{dF_j}{dx} = iK_j F_j - 2i\sum_{pq} T_{j,p,q}\ F_p F_q\ \Big|_{f_j = f_p + f_q} + i\sum T_{j,-p,q}\ F_p^* F_q\Big|_{f_j=-f_p+f_q};\qquad (2)$$

with $F_j = a_j C_j^{1/2} e^{i\int k_j dx};\quad K_j = k_j + i\kappa_j;\quad \sigma_j^2 = k_j \tanh k_j h,$

where ( )* indicates a complex conjugate, and the interaction coefficient is given by

$$T_{j,\pm p,q} = \left(\pm 2k_p k_q + \sigma_p^2\sigma_q^2 + k_p\frac{\sigma_q}{\sigma_j} \pm k_q\frac{\sigma_p}{\sigma_j} \mp \sigma_j^2\sigma_p\sigma_q \right)\frac{\sigma_j\sqrt{g}}{\sigma_p\sigma_q}\left(C_j C_p C_q\right)^{1/2}.\qquad (3)$$

The first sum is taken over "sum" interaction triads, i.e. triads which satisfy the condition $f_j = f_p + f_q$; the second sum is over "difference" triads, $f_j = - f_p + f_q$. For mode j, a_j is the complex wave amplitude, C_j the modal group velocity, h the local depth, k_j the wave number, κ_j the attenuation coefficient, and $|F_j|^2$ is proportional to the modal energy flux. In the absence of nonlinear wave coupling (e.g. T=0), the evolution is linear with each frequency mode obeying the linear dispersion relation and decay rate $\kappa(f)$. The effects of dissipation on nonlinear wave evolution simulated based on the equations (2) and the dissipation curve shown in Figure 1 are illustrated

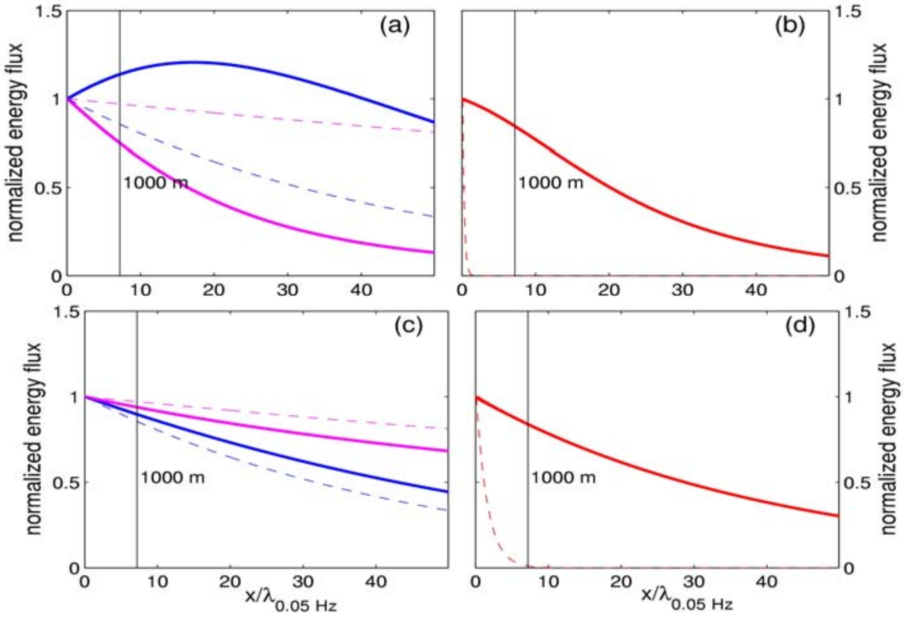

Fig. 2 Evolution of modal flux (normalized by initial value) versus distance in units of mode 1 wavelength (solid lines). Frequencies are f_1 =0.05 Hz (red), f_2=0.5 Hz (blue), and f_3=0.55 Hz (magenta). Here, the water depth is 5 m, fluid mud depth is 30 cm, and mud density 1,130 kg/m^3. Transmission factors at x=1,000 m are (a,b) (< 1%, 93%, 99%), and (c,d) (10%, 93%, 99%) for f_1, f_2, and f_3, respectively. Linear long wave dissipation $\kappa(f_1)$ is more than ten times larger in (a,b) than (c,d). Dashed lines represent the linear (i.e. non-coupled modes) evolution.

by numerical simulations using a single-triad and a full spectrum.

Single-triad runs are performed for a "difference" triad with frequencies f_1=0.05 Hz, f_2 = 0.5 Hz, and f_3 = 0.55 Hz, with initial mode 2-3 amplitudes $a_2 = a_3 = 0.1$ m, propagating over 5 m depth of water and a flat bottom. The frequencies and depths are representative of field conditions consisting of short wind wave groups and a bound long wave. The initial long wave amplitude a_1 is computed as a bound wave:

$$F_1\big|_{x=0} = \frac{2T_{1,-2,3}}{K_1 + K_2^* - K_3} F_2^* F_3 \qquad (4)$$

The short waver frequencies are located on the weak attenuation region of the □-curve shown in Figure 1. The amplitude attenuation for modes 2 and 3 is weak (93% and 99% transmission at 1,000 m, respectively). In contrast, the linear long wave attenuation is strong (transmission of less than 1% at 1,000 m, Figure 2a,b; 10% in Figure 2c,d).

Evolution with nonlinear 3-wave interactions (solid lines) deviate from the linear evolution (dashed lines). With high-frequency modes 2 and 3 having comparable strong transmission, energy is pumped from the shortest mode (3, magenta) which decays faster than linearly, into the middle mode (2, blue), which decays slower than linearly (Figure 2a,c). Compared with linear evolution, the low frequency mode 1 is sustained for a longer distance (Figure 2b,d) by energy transferred from the shorter modes, consistent with a steady energy flux directed toward long waves. Different dissipation rates for the long wave affect the short waves (Figure 2a,c), while the nonlinear rate of decay of the long wave appears to be very similar - slightly slower in the case of weaker long wave dissipation. The rate of energy loss in the long wave band is apparently limited by the strength of the nonlinear flux from the shorter waves. The overall evolution for the total short wave flux is illustrated in Figure 3

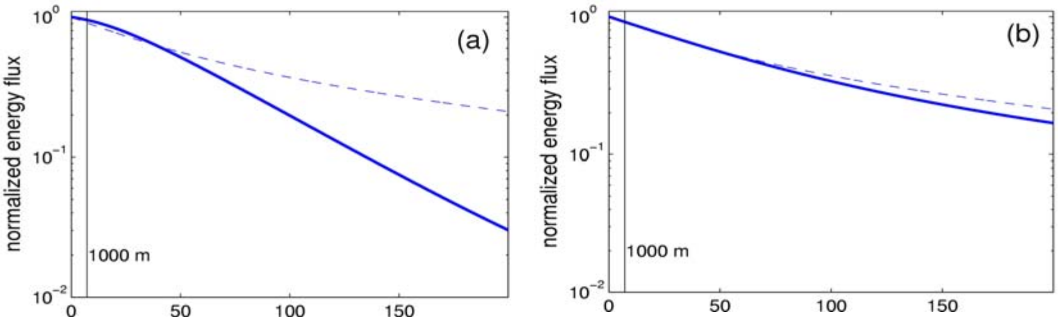

Fig. 3. Total (mode 2 plus mode 3) short wave energy flux (dashed line, linear and solid line, nonlinear). (a) Same parameters as Figure 3a,b. (b) Same parameters as in Figure 3c,d.

for the two cases discussed above. A stronger long wave dissipation rate appears to drain the short wave energy faster.

Numerical results for a full-spectrum simulation are presented in Figure 4. The characteristics of the environment are the same as in the single-triad runs: a flat bottom, 5 m water depth and 30 cm mud depth, fluid-mud density 1,130 kg/m^3 and dynamic viscosity of 1,000 Pa s. The spectrum is characteristic of locally forced short wind waves in deep water, a JONSWAP spectrum with a peak period of T_{peak}=4 s, and significant height of H_{sig}=1 m. Figure 4 represents the evolution of the energy flux spectrum, normalized by the initial spectrum. In the absence of dissipation, linear evolution conserves modal spectrum. Figure 4a shows the effect of dissipation on the evolution of the spectral flux. Long waves are strongly attenuated and disappear from the spectrum within the first 1 km of propagation. Higher frequencies are less affected by the presence of mud. Figure 4b shows the nonlinear (3-wave interactions) evolution of the same spectrum. Mediated by the 3-wave interaction energy exchange mechanism, the strong dissipation in the long wave band is draining energy from modes with f > 0.5 Hz which otherwise would experience practically no dissipation.

CONCLUSIONS

Nonlinear wave dynamics in cohesive sedimentary environments are poorly understood, in part due to the lack of field and laboratory data. As surface wave motion attenuates exponentially with depth, only long waves have been traditionally assumed to interact with the soft muddy bottom. However, recent observations show unexpected dissipation occurring in the short wave band. These suggest the presence of active nonlinear coupling of short- and long-wave bands which, in combination with a long-wave spectral energy sink, produces net energy transport from short to long waves.

No data are available at this time to test this hypothesis. The numerical experiment presented here examines the combined effect of dissipation with a frequency dependence characteristic of muddy bottoms, and nonlinear quadratic interactions. The results of this simplified model support the assumption that wave nonlinearities can significantly modify wave evolution by transferring energy from the short to the long wave spectral band, where it is efficiently dissipated within the fluid mud layer. They also suggest that the rate of this energy transfer depends on the strength of the dissipative effect and the strength of the nonlinear 3-wave

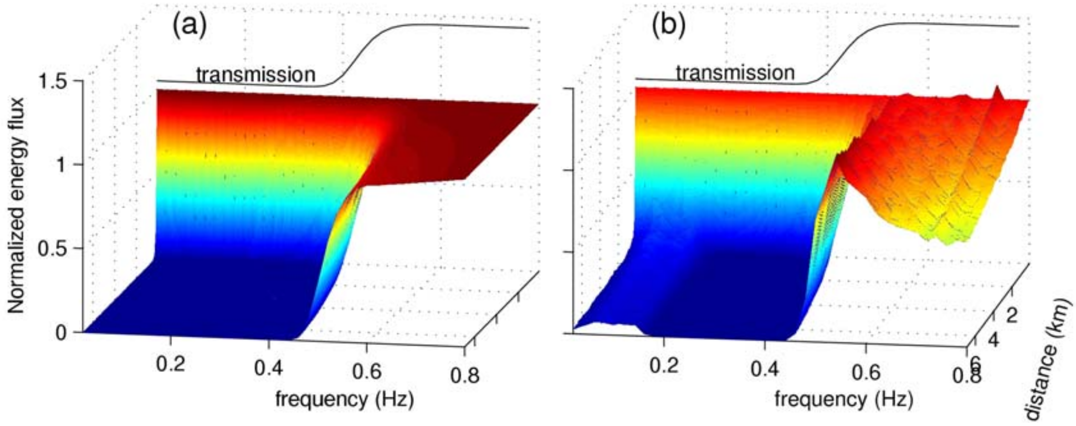

Fig. 4. Evolution of a short wave spectrum (JONSWAP, f_{peak} = 0.4 Hz, H_{sig} = 1 m). The shape of the amplitude transmission at 1,000 m as a function of the frequency is sketched above the plots. Spectral flux density is normalized by initial values. The water and mud-layer depth, and fluid-mud dynamic characteristics are the same as in Figure 2.

coupling, and consequently on the dynamic characteristics of the mud, water and mud layer depths, wave energy and its spectral distribution. More modeling, field and laboratory work is needed to study the details of the processes discussed here.

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